QUANTITATIVE STUDIES OF SHOCK-WAVE PROCESSES BEHIND A SHOCK WAVE WITH THE AID OF SHADOW METHODS

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Shadow methods are widely used in the study of aerodynamic and aerophysical processes (see [1–3] and other papers).

In the following, it is shown that a method described in [4] is well suited for investigating inhomogeneities bounded by a shock wave. Babichev has applied a multiplicative method of identifying a singularity of an empirical deflection-angle function to the theoretical analysis of the problem of calculating discontinuous density distributions. However, the use of an additive method of identifying singularities in the solution of the problem of direct determination of ρ behind a compression shock is also free of the requirement of knowing the optical intensity of a jump. In addition, this method makes it possible to avoid placing additional constraints on the nature of the changes in $\partial n/\partial r$ at the edge of a section across an axisymmetric inhomogeneity as compared to those of $\partial n/\partial r$ in the central portions of the cross section.

The apparatus developed (coefficients $\beta_{i,\nu}$ for N = 5, 10, 25, 50) is suitable for calculating the density of gas flows behind a shock wave, provided the number of zones N into which the cross section of the inhomogeneity breaks down is properly selected. We present the results of an investigation of low-density flows behind the exit section of a shock tube.

1. We will show that the method [4] is applicable to the case of a density discontinuity at an inhomogeneity interface (shock wave). It is possible to determine $\rho(r_i) - \rho_0$ directly without predetermining $n^{-1}\Delta n$ in the compression shock and without changing from total deflection angles ε^*, defined by refractive-index gradients within the inhomogeneity and the shock wave, to deflection angles ε, defined only by refractive-index gradients within an axisymmetric inhomogeneity.

It has been shown by Tatarenchik and Skotnikov that in the case of an inhomogeneity bounded by a shock wave, the deflection angle on a shadowgraph $\varepsilon^*(r)$ is equal to the sum $\varepsilon(r) + \theta(r)$. Here, $\theta(r)$ is the deflection angle created by the passage of the beam through the shock front:

$$\theta(r) = 2\frac{\Delta n}{n_0}\frac{r}{(1-r^2)^{1/2}} \tag{1.1}$$

where $\Delta n/n_0$ is the optical intensity of a compression shock at the inhomogeneity interface and n_0 is the value of the reflective index in front of the jump.

In the case of shadow measurements, the Abel integral can be written in the form

$$\frac{\varepsilon^*(r_\nu)}{r_\nu} = 2\int_{r_\nu}^{1}\left(-\frac{1}{n_0}\frac{\partial n}{\partial r}\right)\frac{dr}{(r^2-r_\nu^2)^{1/2}} + \frac{\theta(r_\nu)}{r_\nu} \tag{1.2}$$

With the aid of the relation (1.1), we obtain from (1.2) the expression

$$[\rho(r_i) - \rho_0] = \frac{1}{k\pi}\int_{r_i}^{1}\frac{\varepsilon^*\,dr}{(r^2-r_i^2)^{1/2}} \tag{1.3}$$

The form of (1.3) is invariant with respect to compression shock. Only the boundary condition changes in the presence of a shock wave, since $\varepsilon^*(r) \to \infty$ for $r \to 1$. Insofar as (1.3) does not contain Δn, this method of identifying singularities does not require predetermination of Δn.

We calculated the coefficients $\beta_{i,\nu}$ which, from formula

$$\rho(r_i) - \rho_0 = \frac{1}{k} \sum_{\nu=i}^{2N-1/2} \beta_{i,\nu} \varepsilon_\nu^* \quad (i = 2N-2,\ 2N-1) \tag{1.4}$$

make it possible to determine the density at the left outermost point of the last zone (half-zone) for N = 5, 10, 25, 50.

The density behind the shock wave at the points $r_i < r_{2N-2}$ can be determined from the values of the empirical function $\varepsilon^*(r)$ with the aid of tables of the $\beta_{i,\nu}$ coefficients compiled earlier [4] for N = 5, 10, 25, 50. Calculations showed that essentially none of the $\beta_{i,\nu}$ coefficients vary in the presence of a shock wave. The only change is that for each value of i, the coefficients in front of ε (0.98) are replaced by the new coefficients $\beta_{i,2N-1/2}$ for $\varepsilon^*_{2N-1/2}$. The latter were calculated for N = 5, 10, 25, 50.

The apparatus developed was checked with the aid of trial distributions. We set: $-\partial n/\partial r = kr$, $\Delta n/n_0 = a$. Calculations were performed for k = 0.2, 0.5, 1.0; a = 0.5. The calculated and true $n(r_i) - n_0$ values were in good agreement.

2. The method proposed was checked by applying it to the investigation of the propagation of a shock wave behind the exit section of a shock tube [5]. Use was made of a standard IAB-451 schlieren facility. The deflection angle distribution was obtained with the aid of logarithmic diaphragms and etalon lenses [2,3]. The gas pressure was 5 mm Hg, the velocity of the shock wave in front of the exit section of the tube was $\tilde{u}$ = 3670 m/sec.

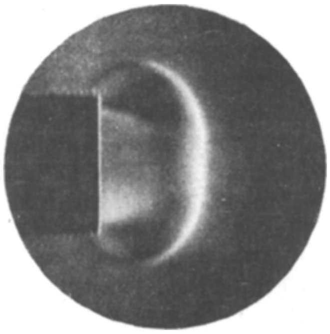

Fig. 1

The shock wave was recorded at the initial instant (12 μsec after it was expelled from the tube), followed by the recording of the flow pattern behind the shock wave, where the boundary of the Mach cone in the center of the flow was distinctly visible (Fig. 1). In its further propagation beyond the exit section of the shock tube, the shock wave is diffracted [6]: diffraction of the first direct shock wave during its propagation leads to the formation of a secondary shock wave behind it; both waves, together with the contact surface, are carried off downstream. This flow pattern (the two waves and the structure of the flow behind them) can be distinctly seen in Fig. 2. Figure 3 shows the deflection angle distribution in the vertical cross section of Fig. 2, located at a distance of 0.8 i.d. from the exit section of the tube. The errors involved in the determination of the deflection angles, assessed on the basis of three independent photometric measurements of the blackening of the photographs at the cross section under consideration, were $\sim 10^{-6}$ (see the angular spread in Fig. 3).

The value of $\rho - \rho_0$ was calculated from the mean values of the deflection angles in the cross section under consideration, with the aid of the coefficients $\beta_{i,\nu}$, breaking down the inhomogeneity radius into 10, 25, and 50 zones. The discrepancy between computations performed for various N was within the limits of the scatter of the $\rho - \rho_0$ values, owing to the determination error of the deflection angles. It is noteworthy that the presence of a region of sufficiently high gradients (caused by the formation of a secondary wave) within the inhomogeneity does not require the use of special procedures for calculating the transition through an internal jump of the angles of deflection. The use of computational procedures with N = 10, 25, and 50 leads to almost identical results. The corresponding distribution $\Delta\rho = [\rho(r_i) - \rho_0]$ is shown in Fig. 4.

The density drop at the inhomogeneity interface (in the shock wave) was $1.3 \cdot 10^{-3}$ kg · sec²/m⁴. This value is smaller than the $\rho - \rho_0$ value calculated in [6] in the case of excitation of translational and rotational degrees of freedom. One of the reasons for the reduction in the density drop obtained may be seen in the substantial decrease in shock wave velocity during propagation beyond the exit section of the tube.

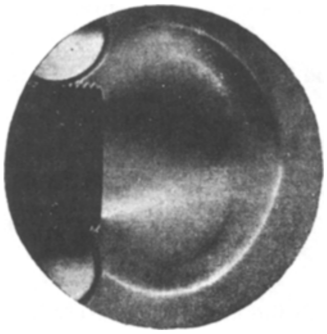

Fig. 2

Behind the shock wave, the density decreases (owing to the formation of the secondary wave), reaching a minimum value at the point $r_i \approx 0.8$. To this value of r_i, there corresponds a maximum of the negative angle of deflection (Fig. 3). Beginning with $r_i \approx 0.45$, there occurs a rapid increase in density, which results in the formation of a Mach cone boundary. At $r < 0.2$, a region of uniform density distribution $\rho - \rho_0 = 8.4 \cdot 10^{-3}$ kg · sec²/m⁴ can be observed. The corresponding evaluation of the equilibrium density yields the somewhat smaller value of $\rho - \rho_0 = 7.5 \cdot 10^{-3}$ kg · sec²/m⁴. This may be attributed to a decrease in the flow temperature, owing to the sudden expansion of the shock wave beyond the exit section of the tube, whereas the pressure hardly varies in this portion of the flow.

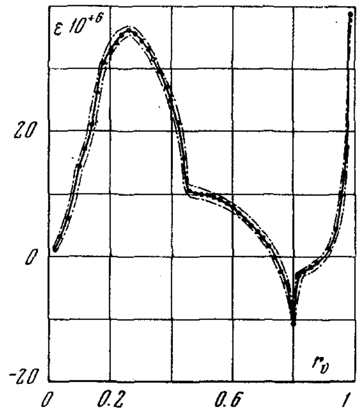

Fig. 3

The investigation showed that the procedure described is well suited for determining densities on the basis of shadow measurements, making use of a computational scheme of the same simplicity as in the case of interferometric measurements.

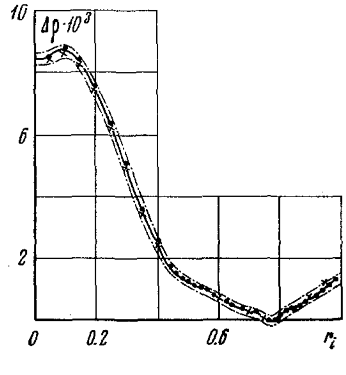

Fig. 4

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